THE SHORTEST SHORT-CUT
A System of Mental Arithmetic

by Oluf Nielsen

The purpose of this pamphlet is to promote an alert mind. As training for the mind, it is helpful in all studies, not only mathematics.

It does not interfere with the current prescribed courses of study and is no criticism of present instruction. It is a helpful addition to a student's mathematical ability.

If a student will memorize the squares of the numbers between 10 and 20 (only nine!), and read this pamphlet, it will give him the power to multiply large numbers such as 175 x 175.

$$\begin{aligned} 175 \times 175 &= (170 \times 180) + (5 \times 5) \\ &= 30{,}625 \\ 275 \times 275 &= (270 \times 280) + (5 \times 5) \\ &= 75{,}625 \end{aligned}$$

The reason for 280 instead of 270 is due to changing the problem to an equation such as:

$$\begin{aligned} 75^2 &= (70 \times 70) + (5 \times 70) + (5 \times 70) + (5 \times 5) \\ &= 4900 + 350 + 350 + 25 \\ &= 5625 \end{aligned}$$

By adding 10 to the second 70, the two center sections of 350 are eliminated and it becomes:

$$\begin{aligned} 75^2 &= (70 \times 80) + (5 \times 5) \\ &= 5625 \end{aligned}$$

All numbers ending in 5 can be squared in this manner.

$$\begin{aligned} 65^2 &= (60 \times 70) + (5 \times 5) \\ &= 4225 \end{aligned}$$

It is fast and easy!

The student will need to memorize the squares of the numbers from 10 to 20.

$10^2 = 10 \times 10 = 100$	$14^2 = 14 \times 14 = 196$	$18^2 = 18 \times 18 = 324$
$11^2 = 11 \times 11 = 121$	$15^2 = 15 \times 15 = 225$	$19^2 = 19 \times 19 = 361$
$12^2 = 12 \times 12 = 144$	$16^2 = 16 \times 16 = 256$	$20^2 = 20 \times 20 = 400$
$13^2 = 13 \times 13 = 169$	$17^2 = 17 \times 17 = 289$	

How to Find the Square of All Numbers to 100

Examples:

$82^2 = 80 \times 80 + (80 + 82)\ (2)$
$= 6400 + 324$
$= 6724$

$81^2 = 80 \times 80 + (80 + 81)\ (1)$
$= 6400 + 161$
$= 6561$

Center $80^2 = 6400$

$79^2 = 80 \times 80 - (80 + 79)\ (1)$
$= 6400 - 159$
$= 6241$

$78^2 = 80 \times 80 - (80 + 78)\ (2)$
$= 6400 - 316$
$= 6084$

Continuing this process:

$77^2 = 75 \times 75 + (75 + 77)\ (2)$
$= 5625 + 304$
$= 5929$

$76^2 = 75 \times 75 + (75 + 76)\ (1)$
$= 5625 + 151$
$= 5776$

Center $75^2 = 5625$

$74^2 = 75 \times 75 - (75 + 74)\ (1)$
$= 5625 - 149$
$= 5476$

$73^2 = 75 \times 75 - (75 + 73)\ (2)$
$= 5625 - 296$
$= 5329$

How to Multiply Any Two Numbers in Any Ten Group

For example (60 to 70):

If the units added = 10:

61 x 69 = (6 x 7) (100) + (1 x 9)
= 4200 + 9
= 4209

62 x 68 = (6 x 7) (100) + (2 x 8)
= 4200 + 16
= 4216

63 x 67 = (6 x 7) (100) + (3 x 7)
= 4200 + 21
= 4221

64 x 66 = (6 x 7) (100) + (4 x 6)
= 4200 + 24
= 4224

If the units added do not = 10, adjust to use units which = 10 and add or subtract the difference.

63 x 69 = 61 x 69 + (2) (69)
= 4209 + 138
= 4347

71 x 77 = 73 x 77 - (2) (77)
= 5621 - 154
= 5467

Multiplying by Comparative Measure Called the Use of the Bar

The bar of any number is the nearest ten to it. For example, the bar of 38 is 40 and the bar of 78 is 80.

If both numbers are *under* their bars, *subtract* a number which is equal to *multiplier plus bar of multiplicand* times *number under the bar* from *product of the two bars*.

For example, 78 is 2 under its bar of 80.

78 (multiplicand) x 38 (multiplier)

78 x 38 = 40 x 80 - (80 + 38) (2)
= 3200 - 236
= 2964

If both numbers are *over* their bars, *add* a number which is equal to *multiplier plus bar of multiplicand* times *number over the bar* to *product of the two bars*.

82 x 42 = 80 x 40 + (42 + 80) (2)
= 3200 + 244
= 3444

79 x 39 = 80 x 40 - (39 + 80) (1)
= 3200 - 119
= 3081

77 x 37 = 80 x 40 - (37 + 80) (3)
= 3200 - 351
= 2849

If the numbers are both over or under the bar but not in equal amounts, adjust to make them equal and add or subtract the difference.

78 x 37 = 77 x 37 + 37
= 2849 + 37
= 2886

If the *multiplicand is under the bar* and the *multiplier is over the bar*, then *add* the difference between the multiplier and the bar of the multiplicand.

78 x 42 = 80 x 40 + (80 - 42) (2)
= 3200 + 76
= 3276

If the *multiplicand is over the bar* and the *multiplier is under the bar*, *subtract* the difference between the multiplier and the bar of the multiplicand.

82 x 38 = 80 x 40 - (80 - 38) (2)
= 3200 - 84
= 3116

A split bar is when one number is over the bar and the other is under.

Examples:

88 x 62 = 90 x 60 + (90 - 62) (2) **Split Bar**
= 5400 + 56
= 5456

92 x 58 = 90 x 60 - (90 - 58) (2) **Split Bar**
= 5400 - 64
= 5336

148 x 92 = 150 x 90 + (150 - 92) (2) **Split Bar**
= 13500 + 116
= 13616

148 x 88 = 150 x 90 - (150 + 88) (2) **Both Under**
= 13500 - 476
= 13024

152 x 92 = 150 x 90 + (150 + 92) (2) **Both Over**
= 13500 + 484
= 13984

Note: Always select the bar closest to the numbers to be multiplied.

How to Multiply Any Number by 33 1/3 or 25

To multiply by 33 1/3, as a short cut, multiply by 100 and divide by 3.

33 1/3 x 84 = 8400 / 3 = 2800

33 1/3 x 87 = 8700 / 3 = 2900

But if you multiply by 33 only, you subtract 1/100 of the answer or 28 or 29 respectively.

33 x 84 = 8400 / 3 - 28
= 2800 - 28
= 2772

33 x 87 = 8700 / 3 - 29
= 2900 - 29
= 2871

33 x 86 = 8600 / 3 - 28
= 2866 - 28
= 2838

Note: Here remainders of 2 from 8600 / 3 were lost and were not needed.

To multiply by 25, multiply by 100 and divide by 4.

25 x 54 = 5400 / 4 = 1350

Mulitiplication by Formula

In high school algebra:

$(4x)\ (2x) = 8x^2$

In college algebra:

The answer equals half the sum squared minus half the difference squared.

$(4x)\ (2x)$	$= (6x / 2)^2 - (2x / 2)^2$ $= 9x^2 - x^2$ $= 8x^2$
93 x 57	$= (150 / 2)^2 - (36 / 2)^2$ $= 75^2 - 18^2$ $= 5625 - 324$ $= 5301$
766 x 534	$= (1300 / 2)^2 - (232 / 2)^2$ $= 650^2 - 116^2$ $= 422{,}500 - 13{,}456$ $= 409{,}044$
$75{,}500^2$	$= (75 \times 76 \times 1{,}000{,}000) + (5 \times 5 \times 10{,}000)$ $= 5{,}700{,}000{,}000 + 250{,}000$ $= 5{,}700{,}250{,}000$
75,500 x 80,500	$= (156{,}000 / 2)^2 - (5{,}000 / 2)^2$ $= 78{,}0000^2 - 2{,}500^2$ $= 6{,}084{,}000{,}0000 - 6{,}250{,}000$ $= 6{,}077{,}750{,}000$

When the student becomes inquisitive enough he will wonder what put the planets like our earth in orbit at 93,000,000 miles from the sun. What keeps it from being drawn into the sun?

If the north pole was not pointing towards the north star, would we have the four seasons?

At what speed do we travel to make the orbit in 365 1/4 days?

What would happen if the earth did not roll over 1,000 miles per hour establishing day and night divided in the 24 hours?

Would it work if it rolled at 500 miles per hour? Or would the nights be too cold and the days too hot for vegetation?

Wishing to know is the start of thinking. Only the drive to know will start the process. Out of more than 6,000,000 brain cells, some must be inactive. Let us call it cold storage.

Maybe with this mental arithmetic, we can get some of those cells out of cold storage!

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