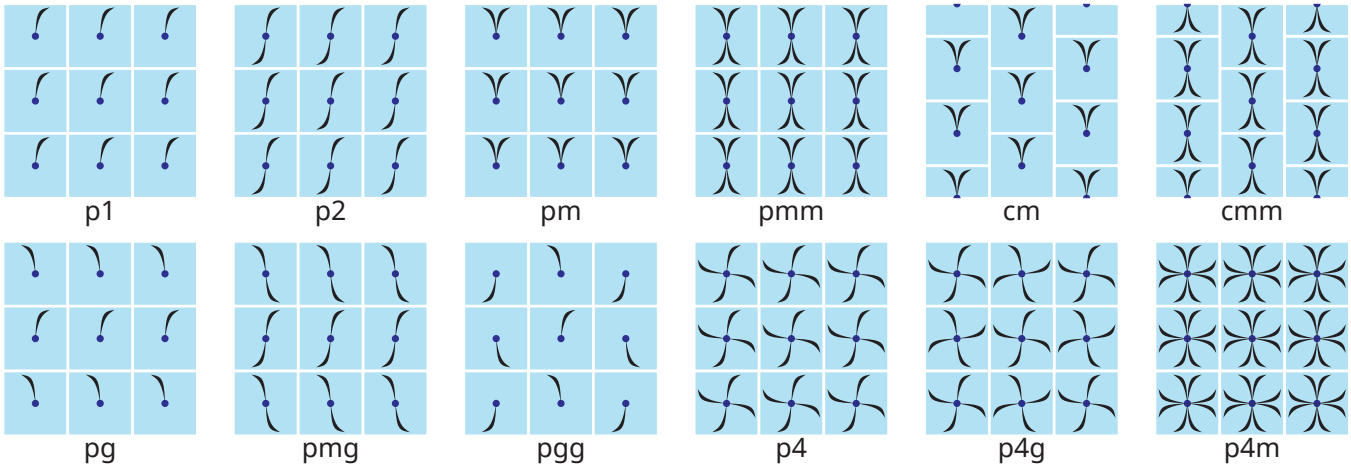
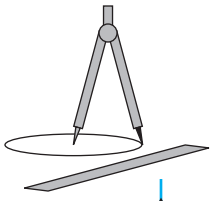
Happy 17th Birthday

Plane Symmetry Groups

There are **17** wallpaper symmetry groups.

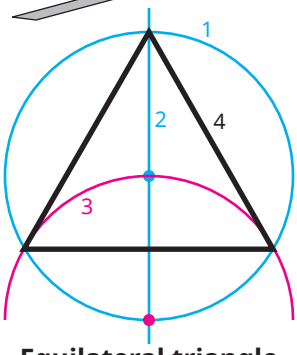


Polygon Constructions

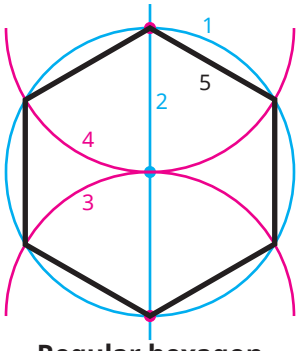


Construct regular polygons using only compass and straightedge.

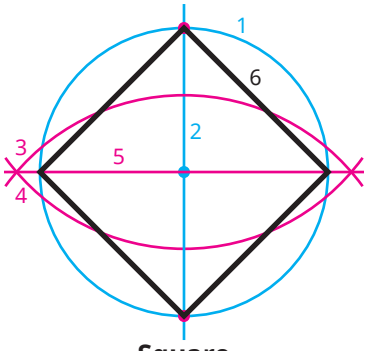
Each construction below begins by drawing a circle (1) and a diameter line (2).



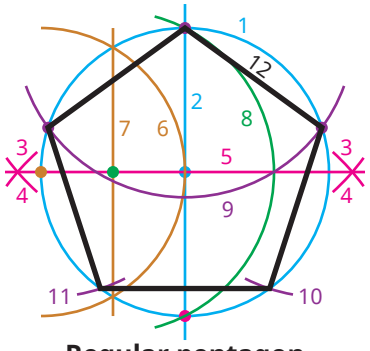
Equilateral triangle
Draw an arc (3) with the same radius as the circle.



Regular hexagon
Draw two arcs (3, 4) with the same radius as the circle.



Square
Draw two arcs (3, 4) to make a perpendicular bisector (5).



Regular pentagon
This construction has a lot of steps!

Some regular *n*-gons are constructible; others are ~~not~~.
n = 3 4 5 6 ~~7~~ 8 ~~9~~ 10 ~~11~~ 12 ~~13~~ ~~14~~ 15 16 17 ~~18~~ ~~19~~ 20 ...

The **17**-gon is constructible, but the 7, 9, 11, 13, 14, and 18-gons are not. How curious! Why is 17 special?
17 is special because it is a Fermat prime. A Fermat prime is a Fermat number $2^{2^m}+1$ that is prime.
For example, $2^{2^2}+1 = 2^4+1 = 16+1 = 17$. The five known Fermat primes are 3, 5, 17, 257, and 65537.
An *n*-gon is constructible if and only if *n* is the product of a power of 2 and/or distinct (unequal) Fermat primes.
For example, *n* = 3, 2^2=4, 5, 2×3=6, 2^3=8, 2×5=10, 2^2×3=12, 3×5=15, 2^4=16, 17, 2^2×5=20, ...

CC BY-SA © 2026 Keith Enevoldsen thinkzone.wlonk.com