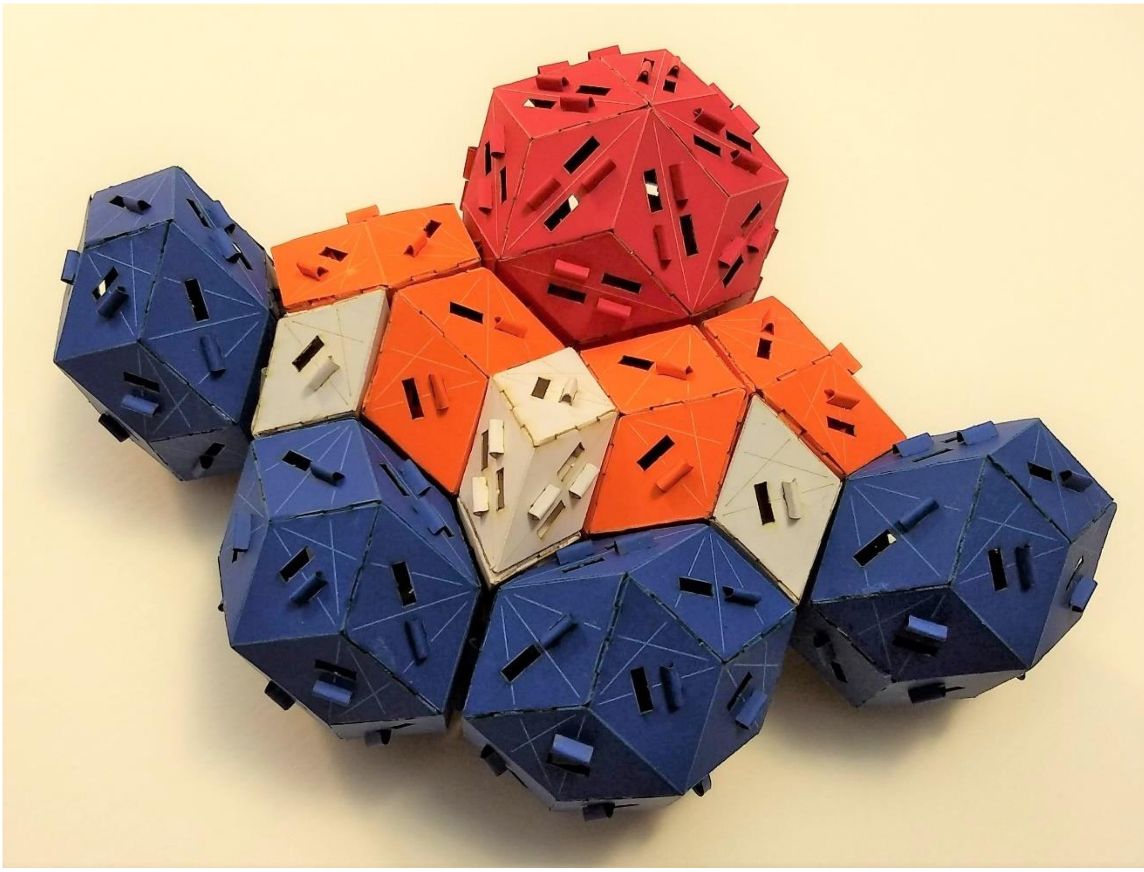
Quasicrystal Blocks: Description and Cut & Fold Instructions

Keith Enevoldsen 2021-05-17



based on Joshua Socolar and Paul Steinhardt,
“Quasicrystals II: Unit-cell Configurations”, Phys. Rev. B34 617-647 (1986).

Contents

1 Introduction

This document contains a description of the quasicrystal building blocks and instructions for making the blocks from cardstock by laser cutting and folding. You can use these blocks to build small quasicrystal assemblies. These cardstock blocks are suitable for building small assemblies; they are not suitable for building large assemblies.

Keith Enevoldsen with cardstock quasicrystal blocks (2021)

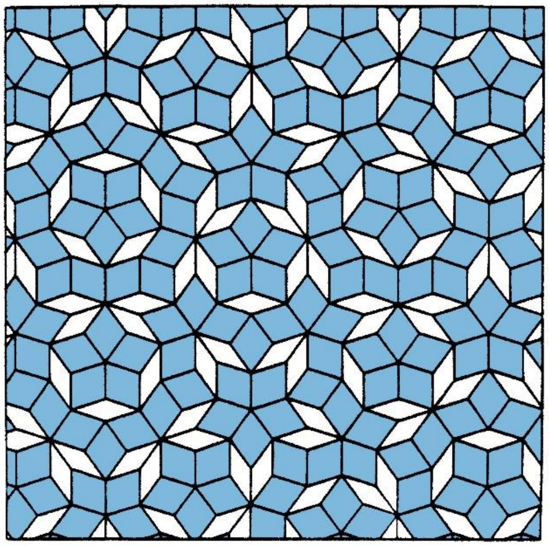
Cardstock quasicrystal blocks

Detailed design of the quasicrystal blocks is based on two articles published in 1986, one by Levine and Steinhardt [QC1] and the second by Socolar and Steinhardt [QC2]. I was inspired by Steinhardt's book, *The Second Kind of Impossible* [Ste19], which contains photos of Steinhardt and Socolar's plastic blocks. See the references at the end of this document.

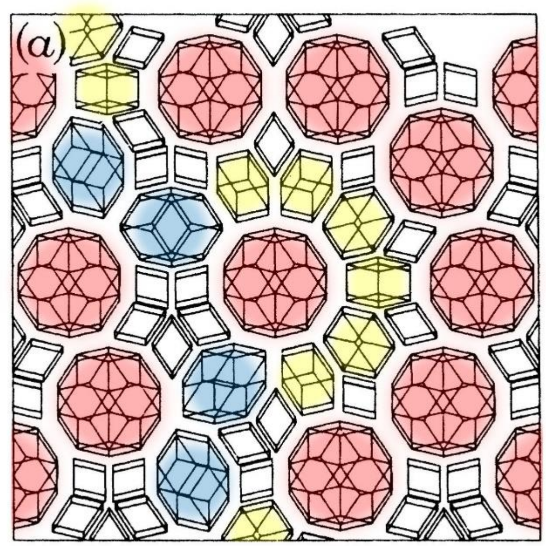
2 Description of Quasicrystal Blocks

2.1 What Are Quasicrystals?

A quasicrystal (quasi-periodic crystal) is a structure, real or abstract, that is ordered but not periodic. The building blocks (unit cells) repeat quasi-periodically, not periodically. You can find a more complete and precise definition of quasicrystals elsewhere (such as [QC1] and [QC2]). Quasicrystals can have unexpected symmetries; for example, icosahedral quasicrystals have a kind of five-fold symmetry.



2D Penrose tiling.
Figure 2 from [QC1] (color added).



Layer of a 3D icosahedral quasicrystal.
Figure 8a from [QC1] (color added).

A 2D quasi-periodic tiling can be built from two or more types of polygons that do not repeat periodically as in an ordinary tiling and yet the tiling has a long-range symmetry. For example, the Penrose tiling with fat and skinny rhombuses has a long-range five-fold symmetry, a kind of symmetry that is impossible in periodic tilings.

Analogously, a 3D quasicrystal can be built from two or more types of polyhedral blocks (unit cells) that do not repeat periodically as in an ordinary crystal and yet the crystal has a long-range symmetry. For example, icosahedral quasicrystals have a long-range five-fold symmetry, a kind of symmetry that is impossible in periodic crystals.

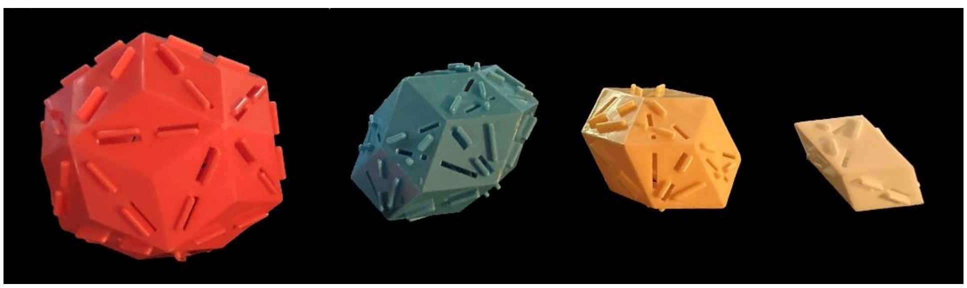
Real quasicrystals, made of atoms, exist. Some are found in nature, others are synthetic.

2.2 What Are Quasicrystal Building Blocks?

[QC1] and [QC2] include a description of a set of building blocks consisting of four types of polyhedra (golden isozonohedra) with local matching rules (using three face decorations) that force the long-range icosahedral symmetry of the quasicrystal.

In 1989, Steinhardt and Socolar designed plastic quasicrystal blocks. These plastic blocks had flanges and slots to enforce the matching rules. They had about a thousand blocks fabricated. The team was trying to understand more about quasicrystal packing and they wanted a physical model to complement their computer models.

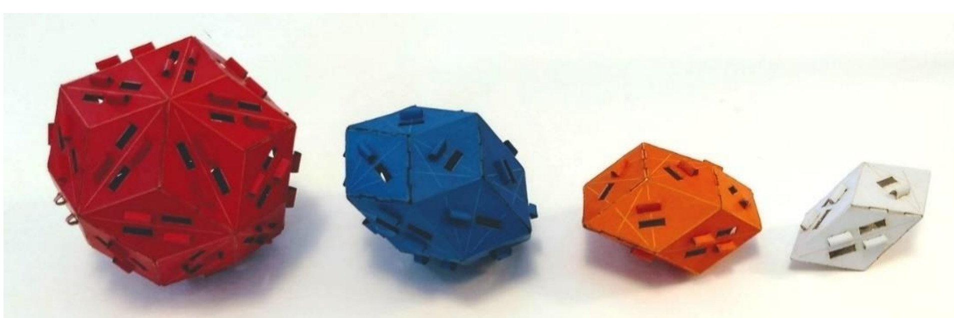
Plastic quasicrystal assembly (photo by Steinhardt)



Plastic quasicrystal blocks (photo by Steinhardt)

In 2020, I (Keith Enevoldsen) saw the photos of the plastic blocks in [Ste19] and I thought there should be some way for quasicrystal fans to get their hands on a small set of connectable quasicrystal blocks. I contacted Paul Steinhardt with the idea of designing patterns for hobbyists to make their own blocks using either a 3D printer or a laser cutter. He encouraged me to design my own patterns and he had many helpful suggestions (such as simplifying the connectors).

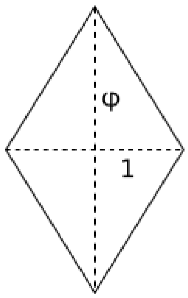
I designed these cut & fold patterns for making quasicrystal blocks from cardstock, including face connectors to enforce the matching rules. The patterns are designed to be cut from cardstock with a laser cutter. Alternatively, you can print the patterns on cardstock and cut them out by hand with scissors and a razor knife.



Cardstock quasicrystal blocks (photo by Enevoldsen)

2.3 Golden Rhombus

Every face of the quasicrystal polyhedral blocks is a golden rhombus, a rhombus whose diagonals are in the golden ratio, $\phi = 1.618$.



Golden rhombus

Property	Value	Equation
Long/short diagonal ratio	1.618	$\phi = (1+\sqrt{5})/2$
Side/short diagonal ratio	0.951	$0.5/\sin(\mathrm{atan}(1/\phi))$
Half acute angle	31.7°	$a = \mathrm{atan}(1/\phi)$
Acute angle	63.4°	$2a$
Half obtuse angle	58.3°	$90 - a$
Obtuse angle	116. 6°	$180 - 2a$

Size of the rhombuses in the cut & fold patterns:

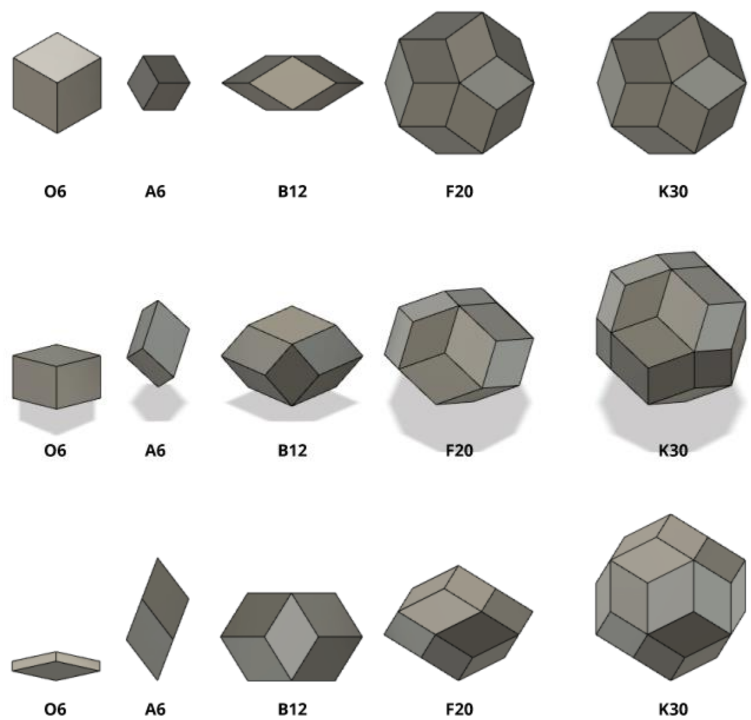
Property	Value	Equation
Short diagonal (width)	30 mm	
Long diagonal (length)	48.54 mm	width × ϕ
Side (edge)	28.53 mm	width × $0.5/\sin(\mathrm{atan}(1/\phi))$

2.4 Golden Isozonohedra

Quasicrystals with icosahedral symmetry can be built from a subset of the five golden isozonohedra, a subcategory of zonohedra (you can look up the definition of zonohedron). Every face of the golden isozonohedra is a golden rhombus.

The five golden isozonohedra are listed here. I use Coxeter's compact alphanumeric zonohedra identifiers.

Polyhedron name	Faces	Zonohedron id
Oblate or obtuse golden rhombohedron	6	O6 (Obtuse)
Prolate or acute golden rhombohedron	6	A6 (Acute)
Bilinski dodecahedron	12	B12 (Bilinski)
Rhombic icosahedron	20	F20 (Fedorov)
Rhombic triacontahedron	30	K30 (Kepler)



Golden isozonohedra, multiple views

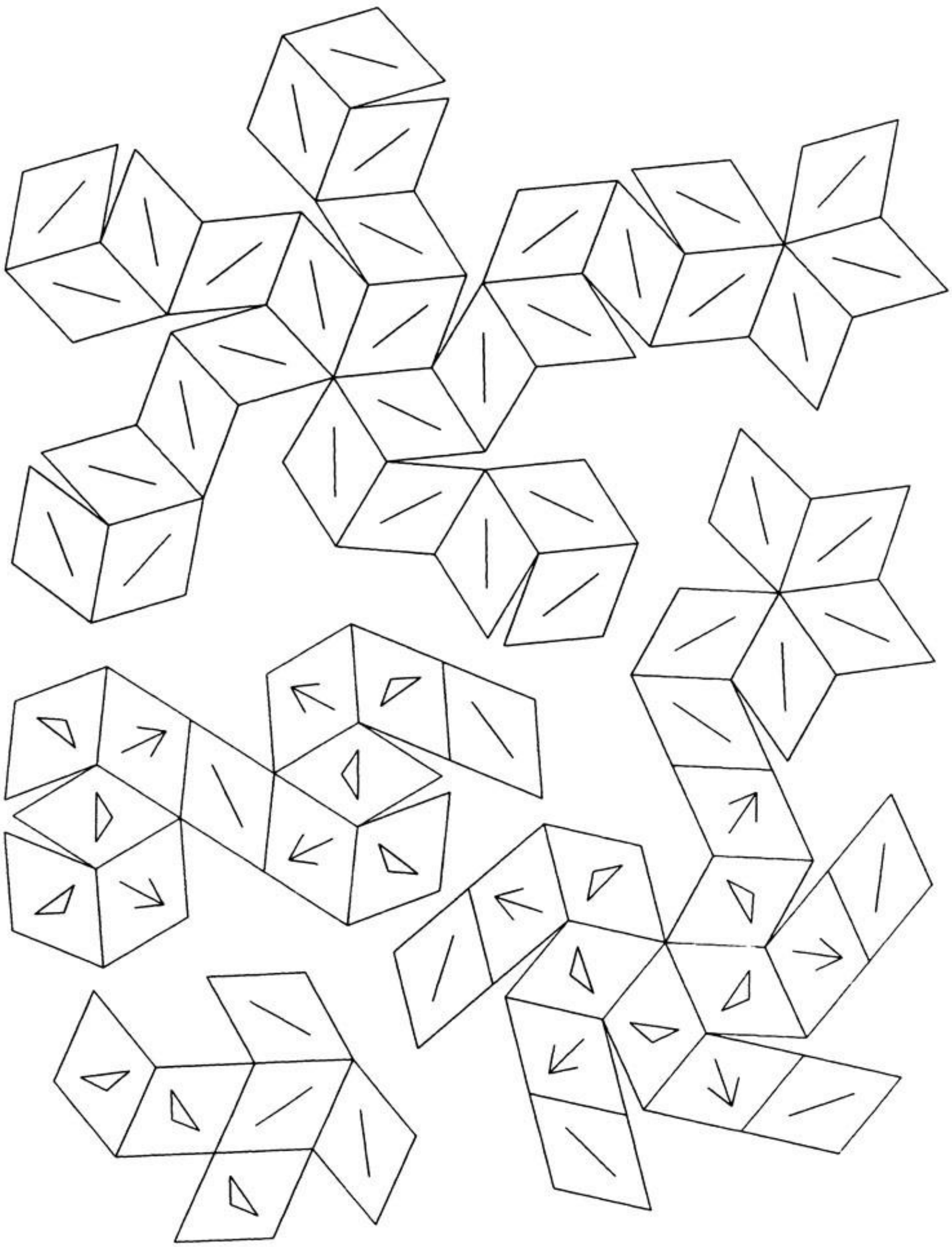
The dihedral angles between adjacent faces of these polyhedra are 36, 72, 108, and 144 degrees (π/5, 2π/5, 3π/5, and 4π/5 radians).

These quasicrystal blocks use four (not all five) of the golden isozonohedra: A6, B12, F20, and K30.

Note: Another way to model quasicrystals uses just two shapes: the two golden rhombohedra (A6 and O6). However, defining matching rules for the two rhombohedra is problematic.

2.5 Face Matching Rules

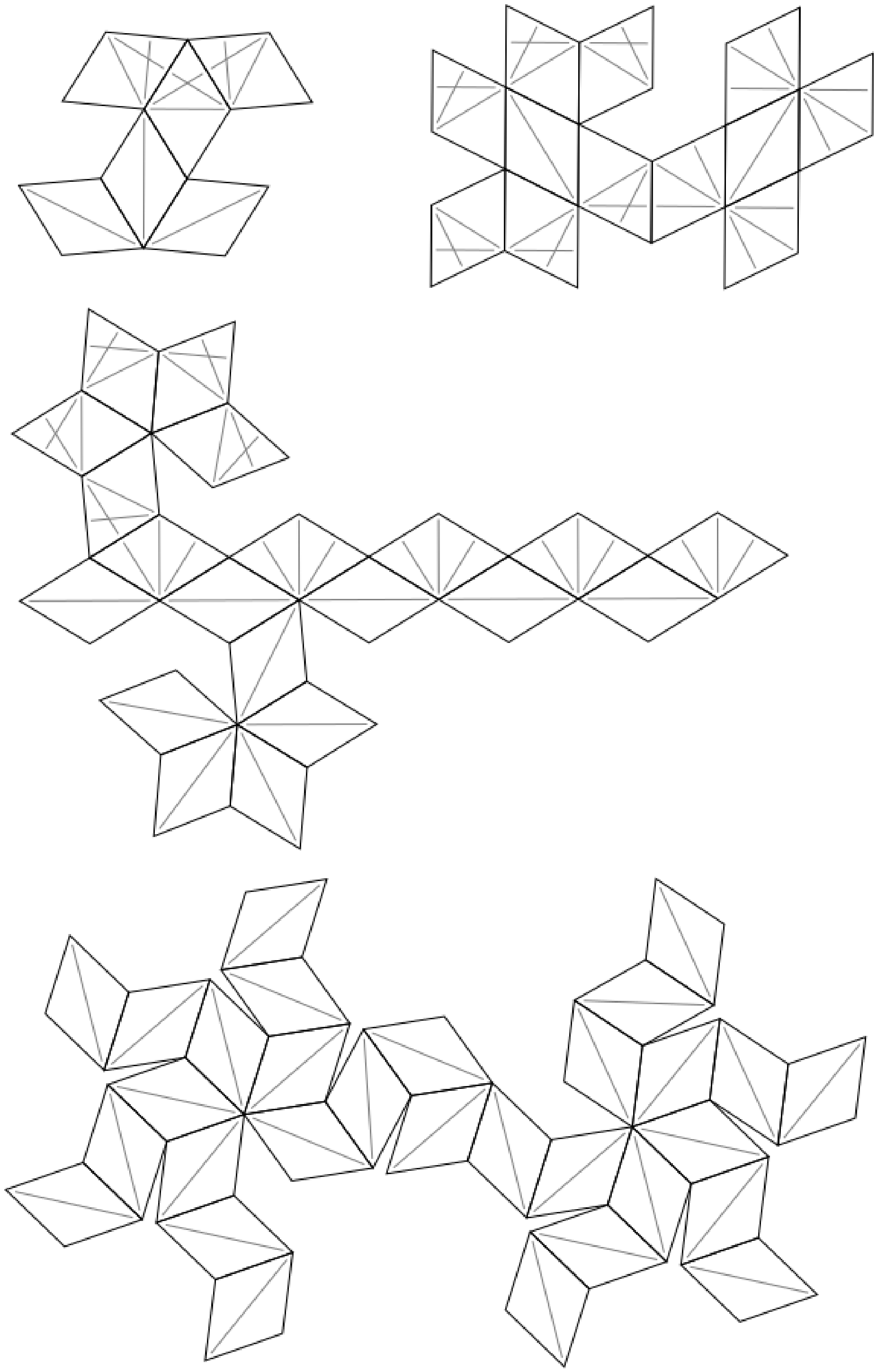
Remarkably, for the four zonohedral blocks (A6, B12, F20, K30), it is possible to define short-range matching rules that force the long-range icosahedral symmetry of the quasicrystal. The faces of the four zonohedra are decorated as shown in the cut & fold pattern from [QC2] (below). The matching rules are simply that adjacent blocks must meet at faces with the same decoration in the same orientation.



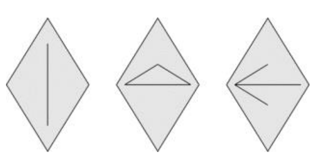
J. Socolar and P. Steinhardt, "Quasicrystals II: Unit-cell Configurations", Phys. Rev. B34 617 (1986), Fig. 13

Cut & fold patterns and matching rule decorations [QC2].
You may print this diagram, then cut & fold to make simple blocks without connectors.

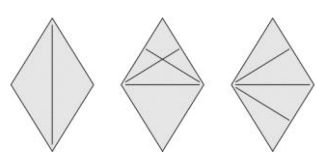
My cut & fold patterns and face decorations are slightly different but equivalent to those in [QC2].



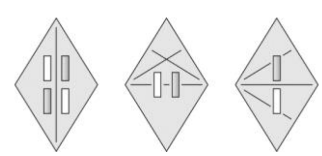
Cut & fold patterns and matching rule decorations (Enevoldsen) equivalent to the patterns in [QC2].
You may print this diagram, then cut & fold to make simple blocks without connectors.



Symbol decorations suggestive of the Ammann lines [QC2]



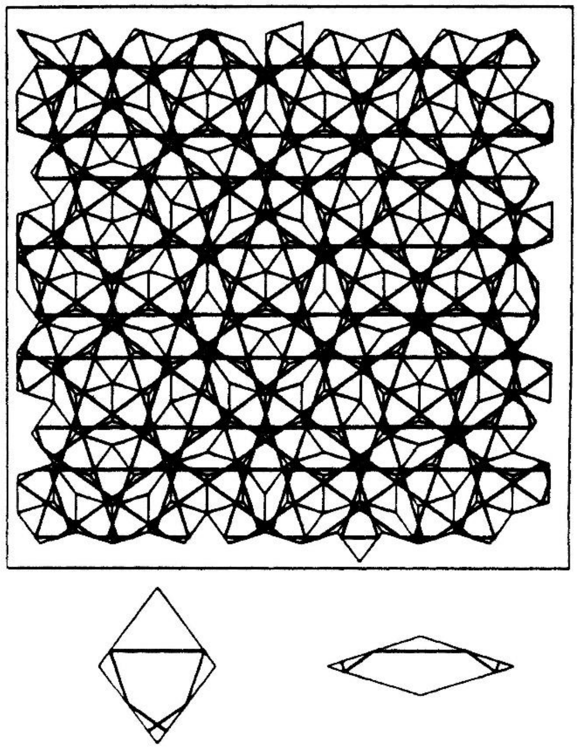
Ammann line decorations



In the connectable blocks, the connectors enforce the matching rules

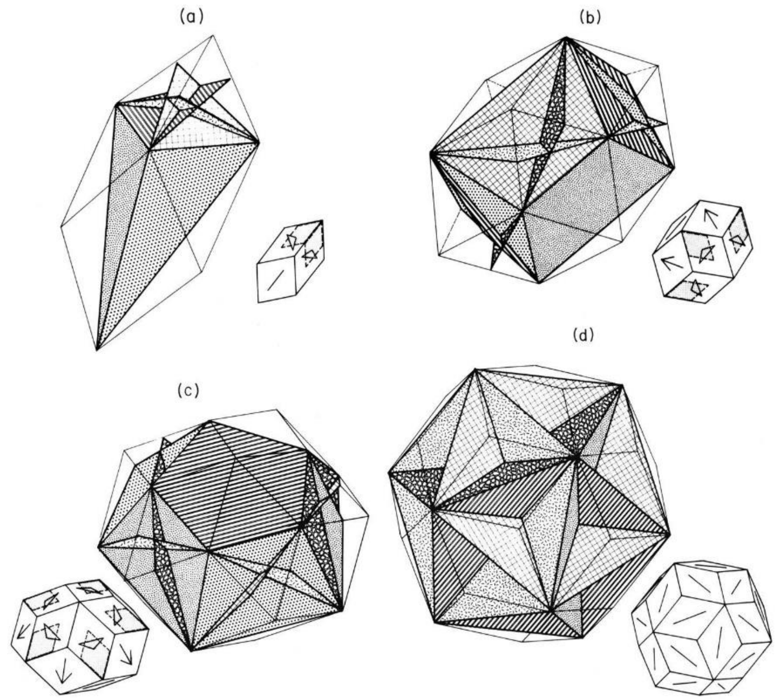
2.6 Ammann Planes

The matching rules are based on Ammann planes. Here is a brief introduction to Ammann bars in 2D Penrose tilings and Ammann planes in 3D quasicrystals. You can read more about Ammann bars and planes in [QC1] and [QC2].



D. Levine and P. Steinhardt, "Quasicrystals I: Definitions and Structure", Phys. Rev. B34 596 (1986), Fig. 4

Ammann bars in a Penrose tiling.
Figure 4 from [QC1].



J. Socolar and P. Steinhardt, "Quasicrystals II: Unit-cell Configurations", Phys. Rev. B34 617 (1986), Fig. 10

Ammann plane decorations of quasicrystal unit cells.
Figure 10 from [QC2].

Some 2D quasi-periodic tilings, such as five- or ten-fold quasi-periodic tilings, have corresponding sets of parallel lines, called Ammann bars, that run throughout the tiling, each set with a different orientation and a quasi-periodic spacing. For a Penrose tiling with pentagonal symmetry, there are five sets of parallel Ammann bars at five orientations. The tiles may be decorated with segments of the Ammann bars. These decorations determine local matching rules that generate the long-range symmetry.

Analogously, some 3D quasicrystals, such as icosahedral quasicrystals, have corresponding sets of parallel planes, called Ammann planes, that run throughout the quasicrystal, each set with a different orientation and a quasi-periodic spacing. For an icosahedral quasicrystal there are six sets of parallel planes at six orientations (corresponding to the six pairs of opposite vertices in an icosahedron). The unit cells may be decorated with planar sections (or the faces decorated with line segments) where they intersect the Ammann planes. These decorations determine local matching rules that generate the long-range symmetry.

The parallel Ammann bars or planes are not evenly spaced; instead, each adjacent pair of bars or planes has one of two separation distances (wide and narrow). The ratio of these two distances is an irrational number and so is the ratio of their frequencies of appearance. In the 2D Penrose tiling and the 3D icosahedral quasicrystal, the two ratios are both the golden ratio.

3 Cutting Instructions

3.1 Cardstock Size, Thickness, Colors

The cut & fold patterns are designed to be laser cut from cardstock. The patterns fit on either A4 or US Letter paper if you print or laser cut them at actual size.

The patterns work with cardstock thicknesses ranging from thin (8 mil) to thick (14 mil) but thick (14 mil) is best. Thick cardstock works well for laser cutting but may be too thick for some printers. To help with shopping, this table shows a range of cardstock thicknesses and weights.

Paper Weight			Paper Thickness		Description
g/m2, gsm	**lb Cover*** (gsm/2.70)	**lb Index*** (gsm/1.81)	**mm**	**mil, point, caliper**	
330 g/m2	122# Cover	182# Index	0.36 mm	14 mil	thick cardstock (recommended)
270 g/m2	100# Cover	150# Index	0.30 mm	12 mil	medium cardstock
216 g/m2	80# Cover	120# Index	0.25 mm	10 mil	medium cardstock
176 g/m2	65# Cover	97# Index	0.20 mm	8 mil	thin cardstock

** US pound weights (lb or #) depend on basis size. Cardstock basis size is usually Cover or Index (not Bond).*

Specifically, I recommend this brand of thick cardstock: PLIKE® 330 gsm, 122# Cover, 14 mil, from Gruppo Cordenons (Italy) and Neenah (US). PLIKE® is currently (2020) available from several online paper stores in various sizes, quantities, and colors.

The colors of the 1989 plastic quasicrystal blocks were: red, blue, yellow, and white. For PLIKE® cardstock, I used these stock colors: red, royal blue, orange (yellow was not an option), and white.

3.2 Numbers of Blocks and Sheets

You will need many more of the smallest A6 blocks than the larger blocks. If you want to build assemblies with five-fold symmetry, consider making at least five of each of the three larger blocks. The table below is an aid for estimating how many blocks of each type to make and how many sheets of cardstock are needed. The table shows the average relative numbers of blocks in a large quasicrystal (from [QC2] equation 28), the number of blocks per sheet (A4 or US Letter) for these patterns, and the numbers of sheets needed for each type of block. For example, if you make five large K30 blocks (5 sheets), you will need about 60 small A6 blocks (15 sheets, 4 blocks per sheet).

Block	Large quasicrystal			
	Percentage of blocks	**Blocks per A6**	**Blocks per K30**	**Blocks per five K30s**
A6	76.4%	1	11.8	58.8
B12	9.0%	0.118	1.39	6.9
F20	8.1%	0.106	1.25	6.2
K30	6.5%	0.085	1	5

Block	Blocks per sheet	Small set		Larger set	
		Blocks	**Sheets**	**Blocks**	**Sheets**
A6	4	8	2	60	15
B12	2	2	1	6	3
F20	1	1	1	5	5
K30	1	1	1	5	5

It takes a lot of time to make even a small number of blocks. Manual cutting may take up to an hour per sheet, whereas laser cutting will take less than 10 minutes per sheet. Manual folding and gluing may take up to an hour per sheet.

3.3 Cut & Fold Pattern Files

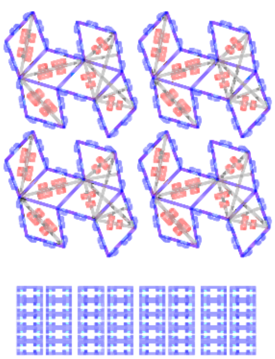
The quasicrystal building blocks cut & fold pattern files may be downloaded from here:

thinkzone.wlonk.com/Quasicrystals/QCBlocks-CutFold.html

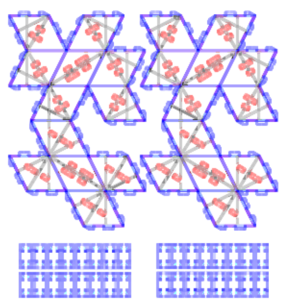
The download includes cut & fold patterns as SVG files for laser cutting and as PDF files for printing.

SVG file for laser cutting	Description	PDF file for printing (A4 size)	PDF file for printing (US Letter)
BlockA6.svg	Block A6 cut & fold pattern	BlockA6(A4).pdf	BlockA6(US).pdf
BlockB12.svg	Block B12 cut & fold pattern	BlockB12(A4).pdf	BlockB12(US).pdf
BlockF20.svg	Block F20 cut & fold pattern	BlockF20(A4).pdf	BlockF20(US).pdf
BlockK30.svg	Block K30 cut & fold pattern	BlockK30(A4).pdf	BlockK30(US).pdf
TestCut.svg	Small pattern for testing your laser cutter settings		

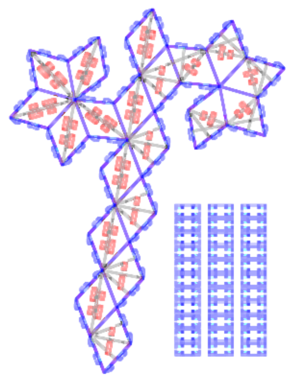
Cut & fold pattern thumbnail images:



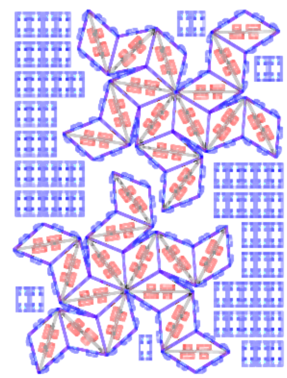
BlockA6.svg



BlockB12.svg



BlockF20.svg



BlockK30.svg

The download also contains instructions for cutting and folding (this document).

3.4 Printing, Manual Cutting and Scoring

Tools: Printer, cardstock, scissors, razor hobby knife.

The cut & fold patterns are intended to be laser cut. But, if you have the patience, you can cut out a few blocks manually.

First, print the PDF cut & fold patterns on cardstock, a different color of cardstock for each type of block. Printable PDF files are provided for both A4 size and US Letter size. Print at Actual Size or No Scaling or Scale 100%, not Fit to Page. To verify that it has been printed at the intended design scale, measure the rhombus dimensions and compare to the size specifications above.

Cut out the patterns manually with scissors and a razor knife. Score the fold lines lightly to make more precise folds.

3.5 Laser Cutting and Scoring

Tools: Laser cutter, cardstock, magnets, scissors, razor hobby knife.

The cut & fold patterns are intended to be laser cut.

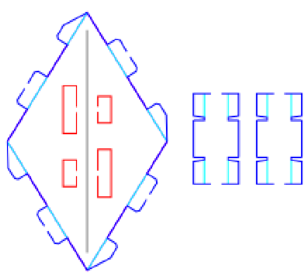
Low power infrared laser cutters (say, less than 60 watts) can cut cardstock, but they may scorch (blacken) the edges of the cuts. High power laser cutters (say, more than 60 watts) can ablate cardstock without any scorching.

The laser cutting bed just needs to be big enough to cut A4 or US Letter size paper.

We assume you know how to use your laser cutter: how to load the SVG pattern file, how to place your cardstock on the cutter bed and hold it flat (with magnets), how to position the cutting pattern over the material, and how to adjust your settings (speed and power) for each type of cut or score.

First, use a small test pattern (such as TestCut.svg) to adjust your settings for cutting and scoring on your chosen type of cardstock. (Note: It is possible you may need different laser settings for different colors of cardstock.)

Several types of lines are used in the pattern, each identified by a different line color. The laser cutter will cut, score, or ignore each color separately, using the cutting/scoring parameters that you specify for your cutter and your material.



Small test pattern with line colors

Line color	Line type	Laser cutter setting
cyan	Folding lines	Scored to be folded
gray	Decoration lines	Scored as a marking
red	Cutting lines for holes	Cut
blue	Cutting lines for outlines	Cut
magenta	Construction lines	Ignore
black	Other lines and text	Ignore

Score the folding lines just deep enough to make a precise fold. Score the decoration lines just deep enough to leave a visible mark. Cut the holes/slots and outlines.

> *Glowforge settings: I used the Glowforge® basic 40-watt CO2 laser cutter in 2020. My speed/power settings (in Glowforge custom units) for PLIKE® were approximately these: cutting 450/100, folding 500/20, marking 500/10.*

After you have adjusted all your laser cutter settings with the small test pattern, laser cut the cut & fold patterns for the blocks, each on a different color of cardstock.

Note: The construction lines are to be ignored (not cut or scored). These are the complete outlines of the rhombuses, the basis for constructing the cut & fold patterns. If you view the SVG file, you will not see these construction lines because the cutting and folding lines are on top of the construction lines everywhere. But when you import the SVG cutting pattern file into your laser cutter software, it will reveal the underlying construction lines (because they are a different color).

The cutting lines have small gaps intended to keep the pieces attached to the sheet as it is being laser cut. This prevents the cut pieces (large and small) from being moved by the airflow caused by the exhaust fan. After laser cutting, use scissors or a razor knife to detach the pieces.

4 Folding Instructions

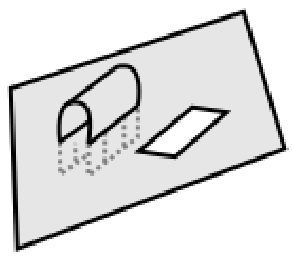
This section gives instructions for constructing the polyhedral blocks.

Constructing the quasicrystal blocks

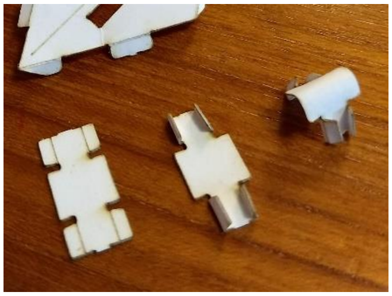
4.1 Constructing the Face Connectors

Tools: Tweezers, razor hobby knife, round toothpicks.

The cut-out face connector are rectangles with tabs at both ends. Fold the sides of each of the two tabs at the scored lines. Bend the middle of each face connector to make a U-shaped half tube with the tabs at the ends of the U. The sides of the tabs should be folded outward. To make the bend, you can either fold the connector loosely without a tight crease or bend it around a round toothpick (or something else that is about 2 mm in diameter).

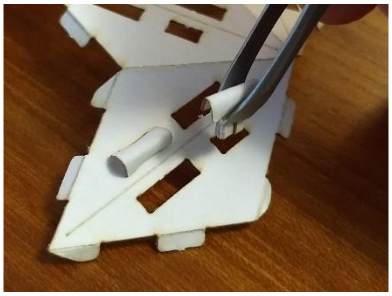


Face connector and face slot diagram



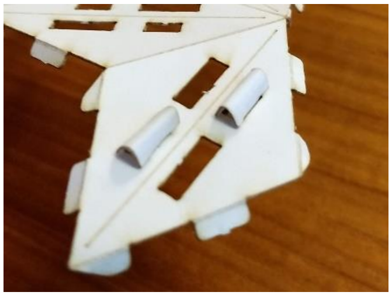
Face connectors cut, folded, and bent

Face connector bent into U-shape



Inserting a face connector into a small slot with tweezers

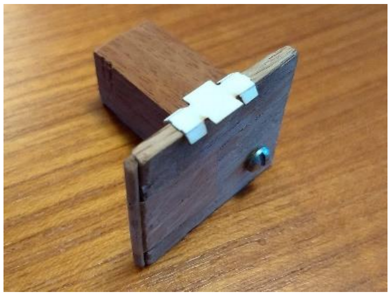
Face connectors anchored on the backside



Face connectors, front side

Tightly fold the sides of each of the two tabs to make the tabs small enough to fit into the small slot in the face. Hold both folded tabs together by pinching them between your finger and thumb or use tweezers. Insert both tabs into the smaller slot in the face. Push the tabs into the front side of the face (the outside face, the decorated side). If necessary, pull them all the way through from the backside (the inside face). Unfold the sides of the tabs on the backside to anchor the connector so it can never be pulled out.

If you are making a lot of connectors, you may want to make yourself a little folding jig to help you fold faster.



Optional folding jig

4.2 Constructing the Polyhedra

Tools for construction: Small sharp tool like razor hobby knife, craft pick, toothpicks (round and/or flat), tweezers.

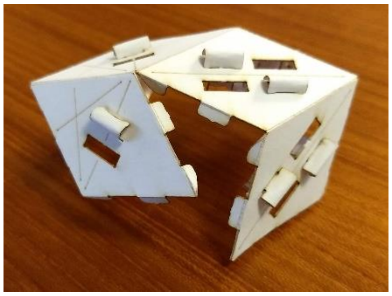
Tools for gluing: Glue for paper crafts, removable painter's tape.

Tools for taping: "Transparent" tape (glossy) or "invisible" tape (matte).

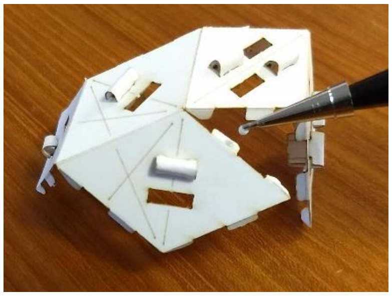
Do not fold up the polyhedron until you have inserted all the connectors into the faces and ensured that they are anchored securely (see above).

Pre-fold all the scored fold lines between the faces and along the edge tabs.

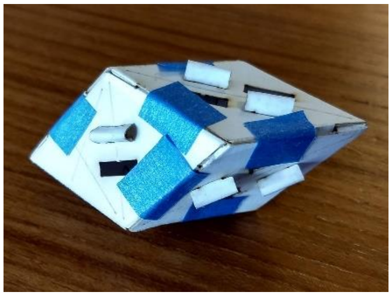
Start folding the faces into a polyhedron, joining one pair of edges at a time, using glue or tape. The edges have interlacing tabs, with the tabs on one edge going between the tabs on the adjoining edge (like interlacing your fingers). All tabs should end up on the inside of the polyhedron. Use a small sharp tool like a hobby knife or toothpicks to poke the tabs inside. It is tricky to get the last face closed with all the tabs on the inside, but you can do it with the help of a small sharp tool. The tabs interlace but they do not lock; glue or tape is required. The interlaced tabs help to hold the edges in place both during and after construction.



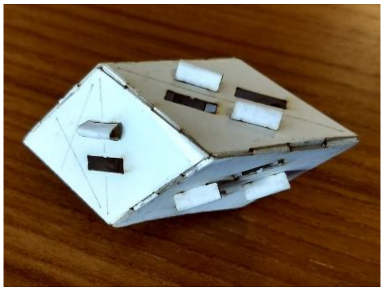
Partly folded to show how the edge tabs will interlace



Gluing the edge tabs



Edge tabs glued and held temporarily with blue painter's tape



Finished glued edges

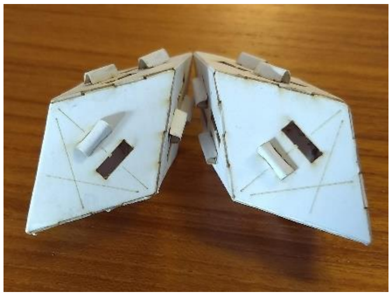
Gluing: The prettiest way to assemble the blocks is to glue the tabs. All tabs should be on the inside. After gluing each edge, tape the edge on the outside with removable painter's tape to hold it until the glue dries. Prepare many small pieces of painter's tape before you begin. When the glue is dry, carefully remove the painter's tape.

Taping: The quickest way to assemble the blocks is to simply tape the edges on the outside, without using any glue. Use "transparent" tape (glossy) or "invisible" tape (matte). All tabs should be on the inside. Prepare many small pieces of tape before you begin.

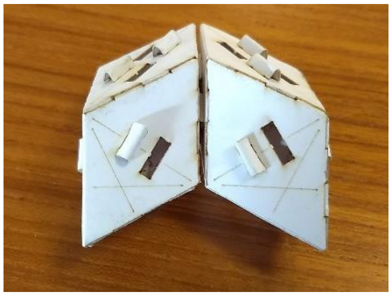
5 Quasicrystal Assembly Instructions

5.1 Joining the Blocks

The quasicrystal blocks are joined by mating the protruding face connectors to the face slots.



Joining blocks



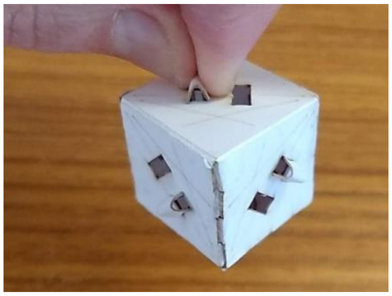
Joined blocks

The three configurations of the face connectors enforce the matching rules. Two faces may be joined only if they have the same face decoration in the same orientation.

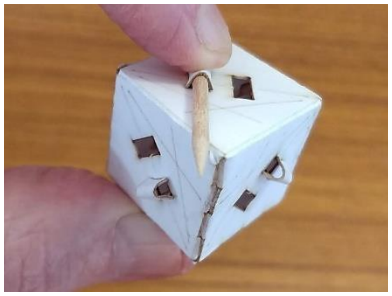
5.2 Adjusting the Fit

Test all your connectors by joining pairs of blocks. The connector tubes should flex a little to grip the sides of the slots. The connectors should not be too loose nor too tight. The connectors should be tight enough to hold two blocks together without outside support, but loose enough that two blocks can be pushed together and pulled apart.

You can adjust the fit of the connectors. If the connector is too tight, make the tube narrower by gently pinching the tube between your finger and thumb. If the connector is too loose, make the tube wider by inserting a round toothpick (or something similar) through the tube and pushing down on the tube with your finger.



Making a connector narrower by pinching



Making a connector wider by inserting a round toothpick

5.3 Building Quasicrystal Assemblies

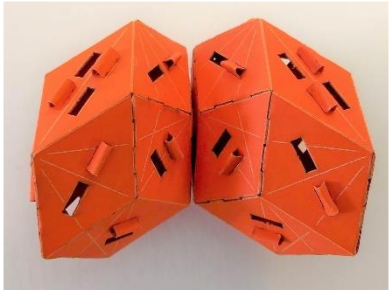
After constructing the blocks, you can build small quasicrystals. A quasicrystal is an assembly of these blocks.

A small quasicrystal assembly

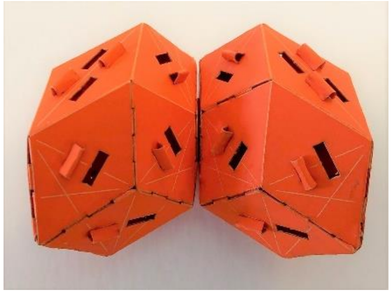
These cardstock blocks are suitable for building small assemblies, typically fewer than 20 or 30 blocks; they are not suitable for building large assemblies. The connectors should be tight enough to hold a small assembly together, but they must be loose enough to wriggle the blocks into position, so they may not be tight enough to hold a large assembly together. In general, you can build assemblies that are small enough to be cradled in your two hands. You can make the blocks fit together more snugly by gently squeezing the assembly between your hands or by putting a long rubber band around the assembly. It is difficult to build larger assemblies because larger assemblies are more fragile.

A quasicrystal must be a contiguous packing of blocks with no empty spaces inside. The face connectors will enforce the matching rules. The matching rules help you to build a true quasicrystal structure, although they do not guarantee it. If your assembly is a contiguous packing and the blocks obey the matching rules, and the blocks on the periphery allow for the contiguous packing to continue and obey the matching rules, then you are likely to have built a true quasicrystal, but you can't really be certain that you will be able to continue adding blocks indefinitely.

During assembly, you will sometimes reach an impasse where it is impossible to fit the next block. If the reason for the impasse is that the space is too tight to wriggle the block into place, then you need to backtrack, disassemble some blocks, and rebuild the same structure, but using a different building sequence. If the reason for the impasse is that the matching rules disallow any block at that location, then you need to backtrack, disassemble some blocks, and rebuild a different structure. For example, compare the two configurations of B12 blocks shown below. The configuration on the left is disallowed because you cannot fit any block into the concavity, whereas the configuration on the right is allowed.



Disallowed configuration

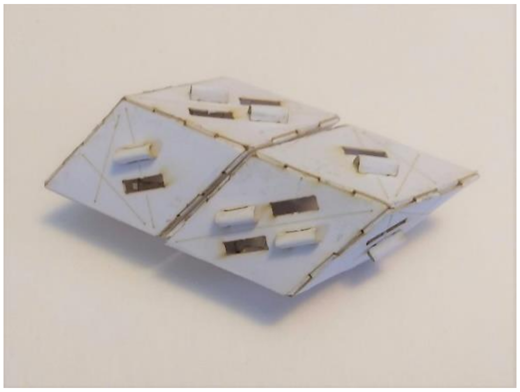


Allowed configuration

5.4 Building Small Quasicrystal Assemblies

This section shows some small quasicrystal structures that you can build with the cardstock blocks.

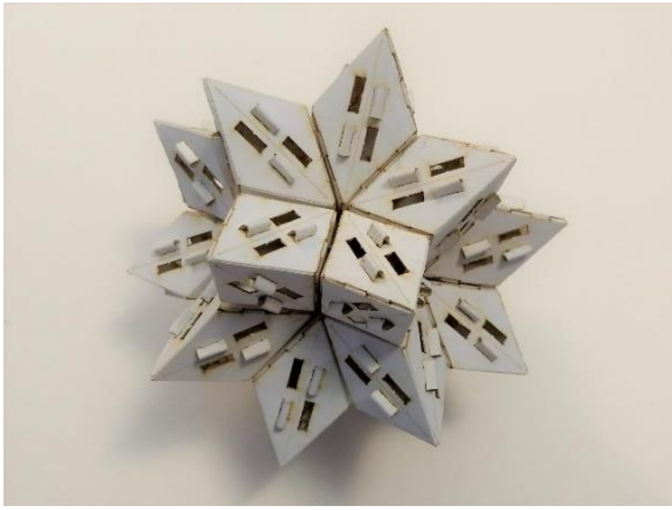
First, you might attempt to build an ordinary periodic crystal with these blocks. You can connect two A6 blocks in a row (below), in what appears to be the beginning of an ordinary periodic crystal, but you will find that you cannot continue this pattern periodically. The connectors, which enforce the matching rules, do not allow you to connect a third A6 block in this same straight row. You will be forced into building a quasi-periodic structure rather than a periodic structure.



Two A6 blocks connected in a row, but you cannot continue this periodically.

You can connect five A6 blocks to make a star with five points. You can connect twenty A6 blocks to make a stellation of a regular icosahedron. You can see how these building blocks produce regular icosahedral structures even though none of the four types of blocks is a regular icosahedron. The concavities in the stellated icosahedron will exactly fit F20 or K30 blocks.

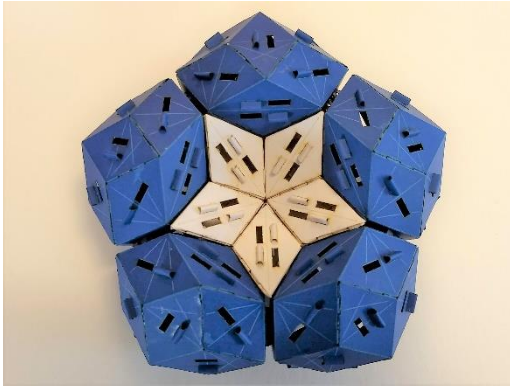
Assembly 1.1. Five A6 blocks making a star.



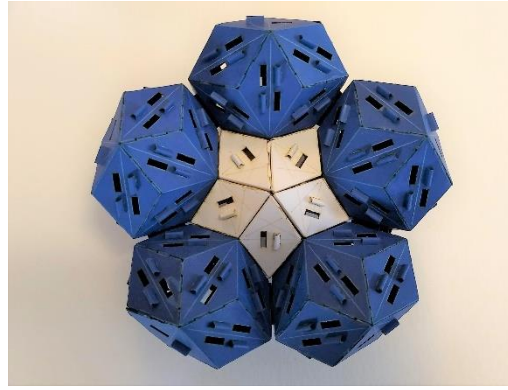
Assembly 1.2. Twenty A6 blocks making a stellated icosahedron. Note: This is in the three-layer diagram (below), spanning all three layers.

Many of the example assemblies shown here have ordinary five-fold rotational symmetry because rotational symmetry is aesthetically pleasing and easy to visualize. But keep in mind that it equally valid to build assemblies without any overall ordinary rotational symmetry. The matching rules will enforce an overall quasi-periodic symmetry regardless of whether the assembly of blocks has any ordinary rotational symmetry.

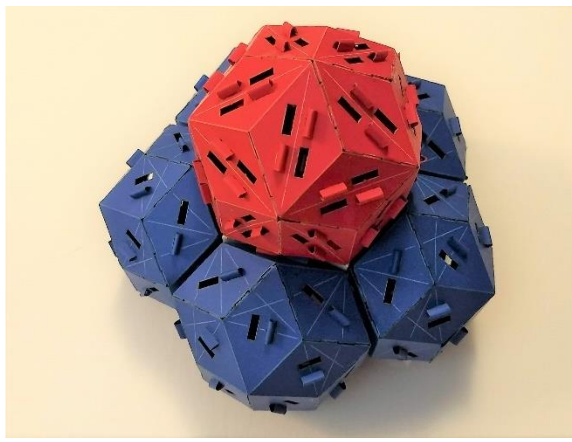
Here are some assemblies that begin with a star of five A6 blocks.



Assembly 2.1. Five A6 blocks surrounded by F20 blocks.
Note: This is on layer 3 of the three-layer diagram (below).

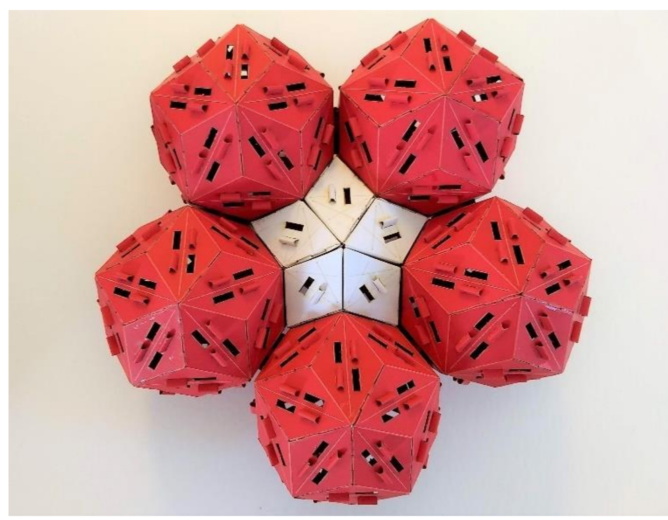


Assembly 2.1. The opposite side.



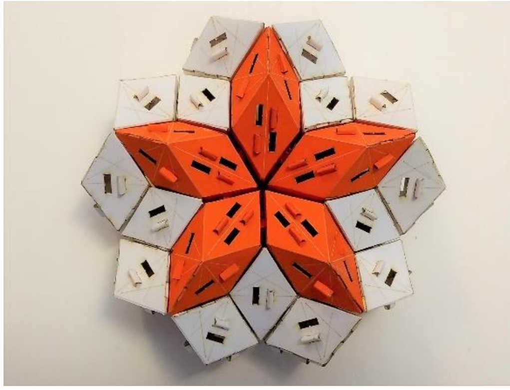
Assembly 2.2. Starting from assembly 2.1, add a K30 block on top.

Assembly 2.3. Five A6 blocks surrounded by K30 blocks.
Note: This is on layer 1 of the three-layer diagram (below).



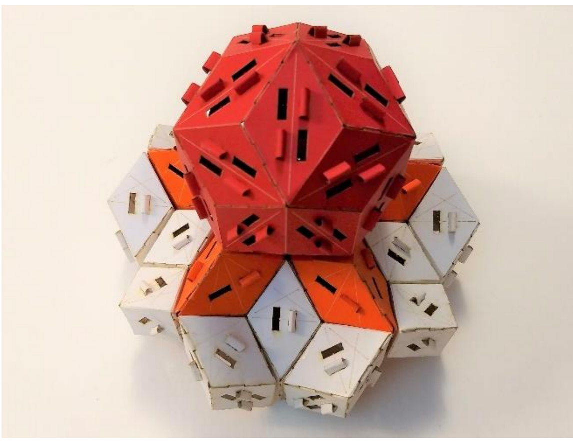
Assembly 2.3. The opposite side.

Here are some assemblies that begin with a star of five B12 blocks.



Assembly 3.1. Five B12 blocks surrounded by A6 blocks.

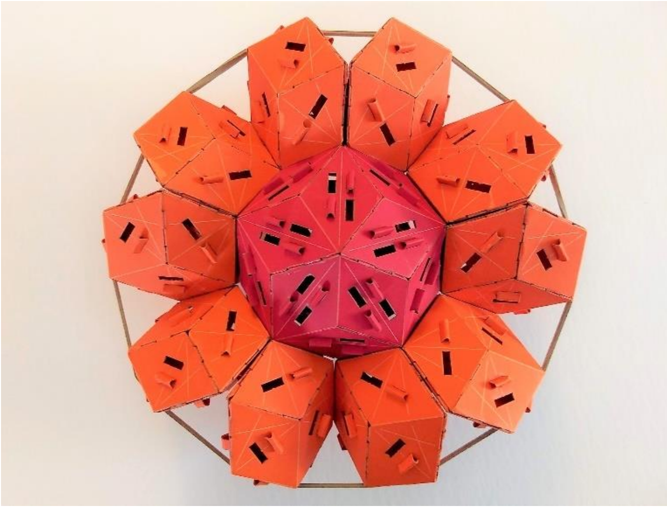
Assembly 3.1. The opposite side.



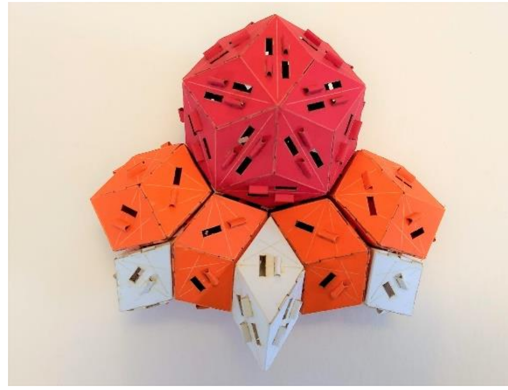
Assembly 3.2. Starting from assembly 3.1, add a K30 block on top.

Assembly 3.3. Starting from assembly 3.1, add a few more A6 blocks, then add a partial ring of F20 blocks and K30 blocks.
Note: This is in the center of layer 3 of the three-layer diagram (below).

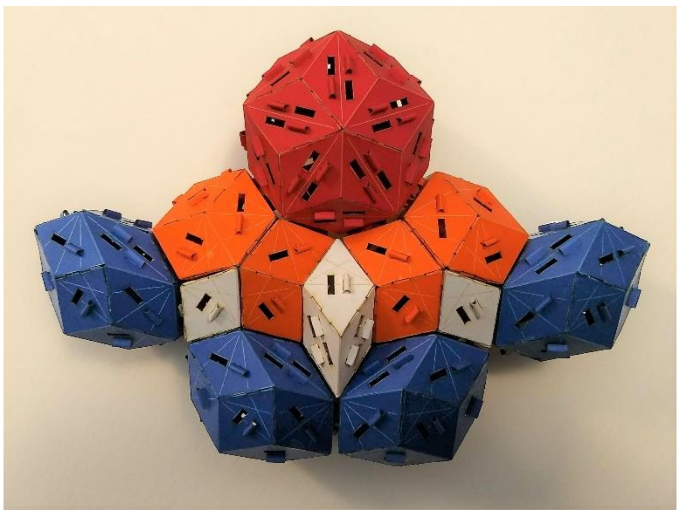
Here are some assemblies that begin with a K30 block in the center.



Assembly 4.1. K30 block surrounded by a ring of B12 blocks. A K30 block may be entirely covered with thirty B12 blocks.



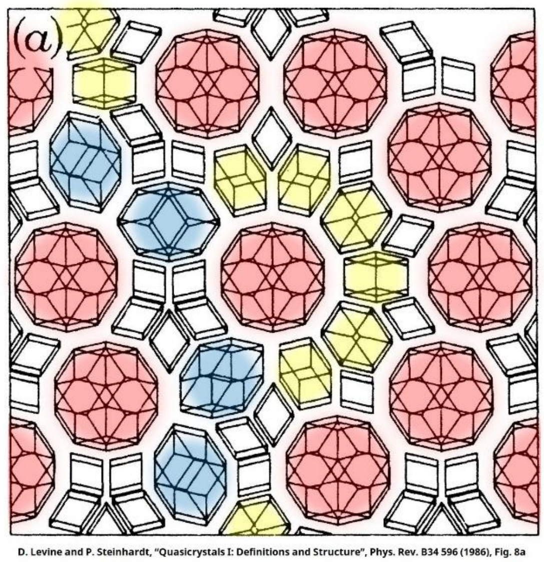
Assembly 4.2. K30 block with a partial ring of B12 and A6 blocks.



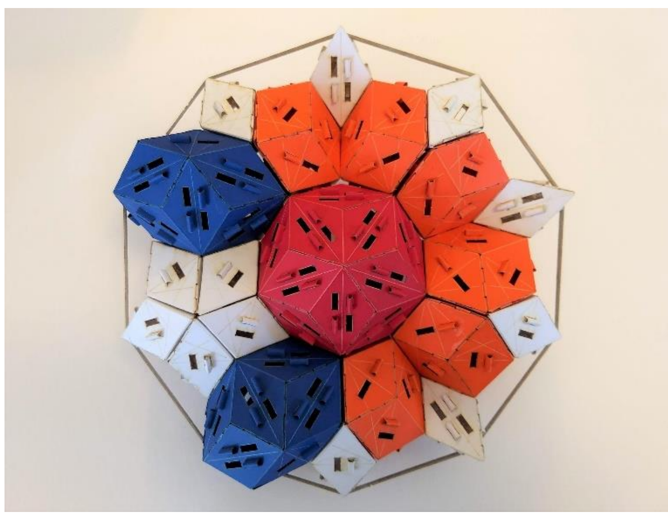
Assembly 4.3. Starting from assembly 4.2, add a partial ring of F20 blocks.

Assembly 4.4. Starting from assembly 4.2, add a partial ring of K30 blocks.
Note: This is in the center of layer 1 of the three-layer diagram (below).

You can use the cardstock blocks to build the central part of this one-layer diagram, figure 8a in [QC1].



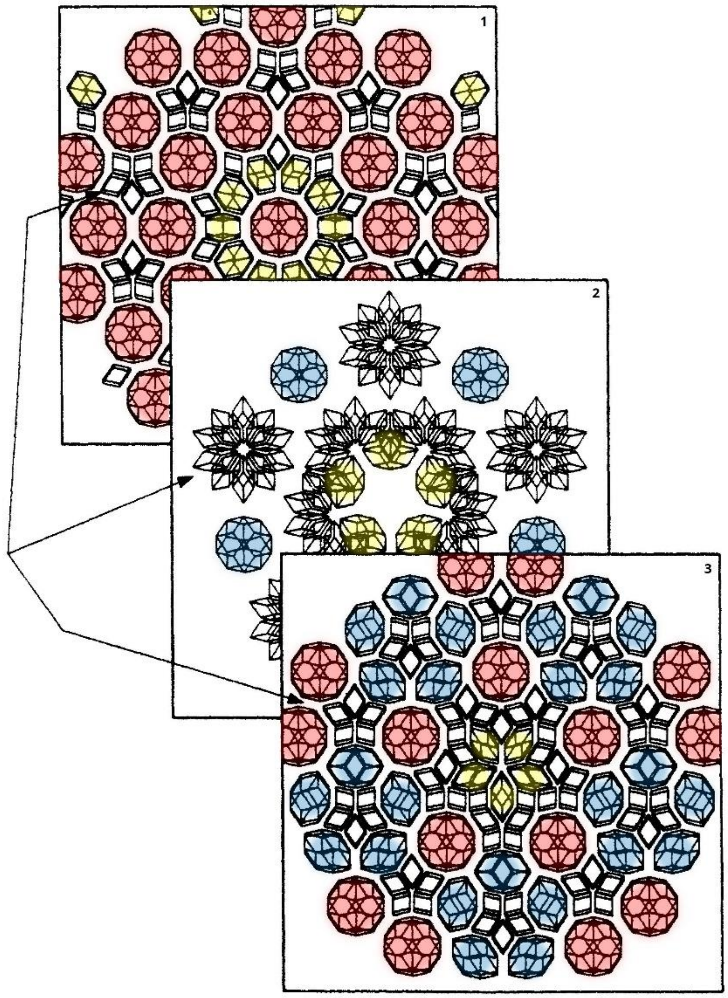
Layer of a 3D icosahedral quasicrystal.
Figure 8a from [QC1] (color added).



Assembly 4.5. Central part of the one-layer diagram.

5.5 Building a Larger Quasicrystal Assembly

You can use the cardstock blocks to build the central part of this three-layer diagram, figure 9 from [QC2].



J. Socolar and P. Steinhardt, "Quasicrystals II: Unit-cell Configurations", Phys. Rev. B34 617 (1986), Fig. 9

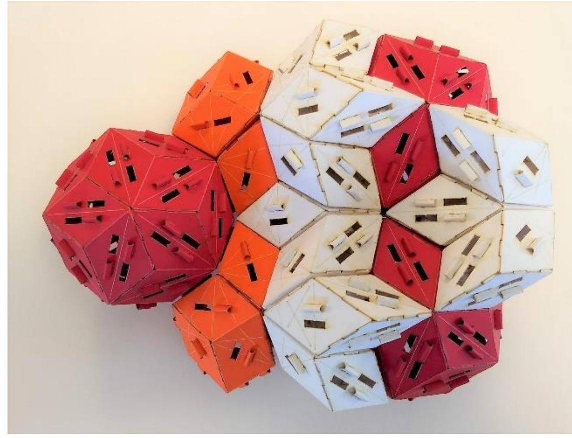
Figure 9 from [QC2] (color added).
Original caption: Thick slice of an icosahedral packing in the PLI class. Three layers of cells are depicted which overlay each other to form the slice. (The unit cells have been drawn as wire frames.) In the 3D packing, a single point is shared by all twenty of the rhombohedra that are indicated by arrows (five in the top frame, ten in the middle, and five in the bottom). The central triacontahedron in the top frame is surrounded by thirty dodecahedra, ten of which are visible in the top frame, five in the middle, and five in the bottom.

The diagram shows three parallel planar layers, but you can also imagine this same structure as concentric spherical layers (like an onion): a central K30 block surrounded by thirty B12 blocks, then surrounded by A6 blocks, then surrounded by K30 blocks and F20 blocks, etc.

The three-layer diagram shows hundreds of blocks, too large to build with the cardstock blocks. But you can build parts of this three-layer structure with the cardstock blocks.

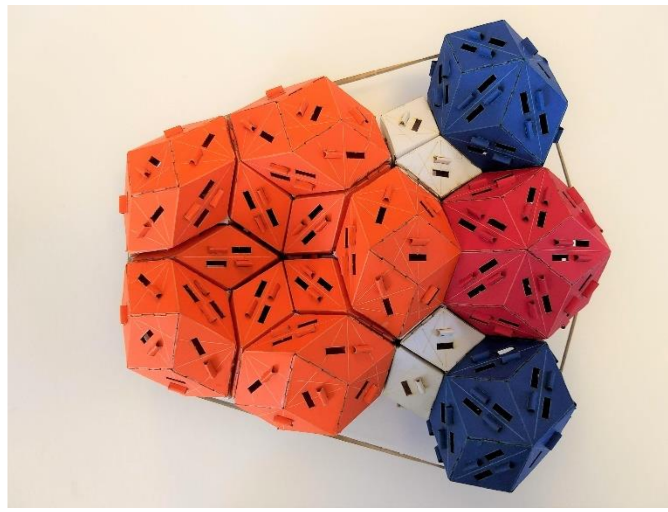
These photos show how to make two assemblies that fit together to make the central part of the three-layer diagram.

*Assembly 5.1. Same as assembly 4.4 (above).
This is in the center of layer 1.*



Assembly 5.2. Starting from assembly 5.1, which is on layer 1, add some A6 blocks from layer 2.

*Assembly 5.3. Same as assembly 3.3 (above).
This is in the center of layer 3.*



*Assembly 5.4. Starting from assembly 5.3, which is on layer 3, add some B12 blocks from layer 2.
Assemblies 5.2 and 5.4 fit together.*

Assembly 5.2 has blocks from layers 1 and 2. Assembly 5.4 has blocks from layers 3 and 2. Joining these would make a bigger assembly with blocks from all three layers. It may be easier to just imagine joining assemblies 5.2 and 5.4, rather than physically joining them. The photos show the mating faces of the two assemblies, so you would need to flip one of these assemblies over to join them together. You can see exactly how all the blocks would connect and obey the matching rules. The hub (leftmost) K30 block in assembly 5.2 will fit into the large nest formed by B12 blocks in assembly 5.4. The K30 block in assembly 5.4 will fit into the large nest formed by A6 and K30 blocks in assembly 5.2.

6 References and Credits

Publications

[QC1] Dov Levine and Paul Steinhardt, "Quasicrystals I: Definitions and Structure", Phys. Rev. B34 596-616 (1986).
First of a pair of groundbreaking scientific papers describing quasicrystal structure.
(It introduces the four quasicrystal blocks.)

[QC2] Joshua Socolar and Paul Steinhardt, "Quasicrystals II: Unit-cell Configurations", Phys. Rev. B34 617-647 (1986).
Second of a pair of groundbreaking scientific papers describing quasicrystal structure.
(It describes the four quasicrystal blocks and matching rules.)

[Ste19] Paul Steinhardt, *The Second Kind of Impossible: The Extraordinary Quest for a New Form of Matter*, Simon & Schuster (2019) (secondkindofimpossible.org).
Steinhardt's story of the discovery of quasicrystals.
(It contains photos of the plastic quasicrystal blocks.)

Websites

Keith Enevoldsen's downloadable cut & fold patterns are here:
thinkzone.wlonk.com/Quasicrystals/QCBlocks-CutFold.html

Paul Steinhardt has an introduction to quasicrystals and archived scientific publications here:
paulsteinhardt.org/quasicrystals

Picture credits

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Photo of Keith Enevoldsen by Julie Enevoldsen.

All other figures and photos are by Keith Enevoldsen.